AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# A-level MATHEMATICS

## Unit Statistics 3

Monday 26 June 2017 Afternoon Time allowed: 1 hour 30 minutes

**Materials**
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
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| Question | Mark |
| 1 | |
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Answer **all** questions.

Answer each question in the space provided for that question.

**1** A machine fills bags with rock salt to be used on paths and driveways.

The weight of salt in a full bag may be modelled by a normal distribution with a mean of $\mu$ kilograms and a standard deviation of $330$ grams.

Calculate the sample size necessary in order that a $98\%$ confidence interval for $\mu$ has a width of at most $200$ grams. Give your answer to the nearest $10$.

**[4 marks]**

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**2** A council wishes to estimate the proportion of library customers who own a hand-held electronic device for reading books.

A sample of $440$ customers showed that $77$ owned such a device.

Stating a necessary assumption about the sample, construct an approximate $99\%$ confidence interval for the proportion of library customers who own a hand-held electronic device for reading books.

**[6 marks]**

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**3** The number of faulty peel & seal envelopes in a pack of $1000$ envelopes produced by a particular company's machine may be modelled by a Poisson distribution.

Prior to a minor upgrade of the machine, the average number of faulty envelopes per pack was $8$ envelopes.

**(a)** Following this upgrade, a randomly selected pack of $1000$ envelopes contained $5$ envelopes that were faulty.

Use an exact test and the $5\%$ level of significance to investigate the success or otherwise of the minor upgrade in reducing the average number of faulty envelopes per pack.

**[5 marks]**

**(b)** The company subsequently decided to carry out a major refurbishment of the machine.

Following this refurbishment, a random sample of $50$ packs of $1000$ envelopes contained a total of $348$ envelopes that were faulty.

Investigate, at the $1\%$ level of significance, whether the major refurbishment of the machine has reduced the average number of faulty envelopes per pack.

**[5 marks]**

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**4** Faults which occur in a particular make of dishwasher can be categorised as mechanical (event $M$), electrical (event $E$) or water (event $W$).

It has been established that $\mathrm{P}(M) = 0.45$, $\mathrm{P}(E) = 0.25$ and $\mathrm{P}(W) = 0.30$.

When a fault occurs, a dishwasher display shows one of three fault codes: $C1$, $C2$ or $C3$.

It is known that:

$$\mathrm{P}(C1 \mid M) = 0.80 \quad \mathrm{P}(C2 \mid M) = 0.05 \quad \mathrm{P}(C3 \mid M) = 0.15$$

$$\mathrm{P}(C1 \mid E) = 0.10 \quad \mathrm{P}(C2 \mid E) = 0.85 \quad \mathrm{P}(C3 \mid E) = 0.05$$

$$\mathrm{P}(C1 \mid W) = 0.00 \quad \mathrm{P}(C2 \mid W) = 0.25 \quad \mathrm{P}(C3 \mid W) = 0.75$$

A dishwasher of this particular make is showing a fault code on its display.
Calculate the probability that this dishwasher:

**(a)** displays fault code $C1$;

**(b)** has a mechanical fault, given that it displays code $C1$;

**(c)** has an electrical fault, given that it displays code $C2$;

**(d)** does not have a water fault, given that it does not display code $C3$.

**[10 marks]**

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**5** The numbers of cars, $X$, and the numbers of bicycles, $Y$, owned by households in a town may be modelled by the following bivariate probability distribution.

| | | **Number of cars ($X$)** | | | | |
|---|---|---|---|---|---|---|
| | | **0** | **1** | **2** | **3** | **P($Y = y$)** |
| **Number of bicycles ($Y$)** | **0** | 0.07 | 0.12 | 0.18 | 0.13 | 0.50 |
| | **1** | 0.03 | 0.18 | 0.07 | 0.02 | 0.30 |
| | **2** | 0.02 | 0.03 | 0.05 | 0.00 | 0.10 |
| | **3** | 0.03 | 0.02 | 0.00 | 0.00 | 0.05 |
| | **4** | 0.00 | 0.05 | 0.00 | 0.00 | 0.05 |
| | **P($X = x$)** | 0.15 | 0.40 | 0.30 | 0.15 | 1.00 |

**(a) (i)** Calculate exact values for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

**[4 marks]**

**(ii)** Given that

$$\mathrm{E}(Y) = 0.85\ ,\ \mathrm{E}(Y^2) = 1.95 \text{ and } \mathrm{E}(XY) = 0.90$$

calculate exact values for $\mathrm{Var}(Y)$ and $\mathrm{Cov}(X, Y)$.

**[3 marks]**

**(iii)** Hence calculate the value of the correlation coefficient between $X$ and $Y$.

**[2 marks]**

**(b)** Calculate values for the mean and the variance of:

**(i)** $T = X + Y$;

**(ii)** $D = X - Y$.

**[5 marks]**

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**6 (a)** The random variable $X$ has a binomial distribution with parameters $n$ and $p$.

Prove that $\mathrm{E}(X) = np$.

**[3 marks]**

**(b)** The random variable $Y$ has a Poisson distribution with mean $\lambda$.

Prove that $\mathrm{Var}(Y) = \lambda$.

**[3 marks]**

**(c)** A company manufactures metal screws. The probability that a screw is faulty is $0.005$. The screws are sold in packets and boxes. The screws in a packet or in a box may be considered to be a random sample.

**(i)** Customer A buys a packet of $50$ screws.

Determine the probability that the packet contains exactly one faulty screw.

**[2 marks]**

**(ii)** Customer B buys a small box of $250$ screws.

Use a distributional approximation to estimate the probability that the small box contains fewer than two faulty screws.

**[3 marks]**

**(iii)** Wholesaler C buys $50$ large boxes, each containing exactly $1000$ screws.

Use a distributional approximation to estimate the probability that the **total** number of faulty screws in the $50$ large boxes is more than $240$.

**[5 marks]**

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**7** Results from an investigation into the characteristics of saltwater crocodiles included the following summarised data on the lengths, $x$ metres, of a sample of $40$ adult males and the lengths, $y$ metres, of a sample of $30$ adult females.

Males: $\sum x = 181.20$ $\sum (x - \overline{x})^2 = 11.7022$

Females: $\sum y = 86.40$ $\sum (y - \overline{y})^2 = 3.4806$

**(a)** Investigate, at the $5\%$ level of significance, the hypothesis that the mean length of adult male saltwater crocodiles exceeds that of adult female saltwater crocodiles by more than $1.5$ metres.

**[9 marks]**

**(b)** Deduce that, for the test of the hypothesis in part **(a)**, the critical value of $\overline{X} - \overline{Y}$ is $1.68$ , correct to three significant figures.

**[2 marks]**

**(c)** It is subsequently established that the mean length of adult male saltwater crocodiles exceeds that of adult female saltwater crocodiles by $1.85$ metres.

Determine the power for a test of the hypothesis in part **(a)** at the $5\%$ level of significance, based upon random samples of $40$ adult male saltwater crocodiles and $30$ adult female saltwater crocodiles.

**[4 marks]**

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**END OF QUESTIONS**